CBCS SCHEME

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15MAT11

First Semester B.E. Degree Examination, Jan./Feb. 2023
Engineering Mathematics – I

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Obtain the pedel equation of $r = a\,\text{sech}\,n\theta$. **(06 Marks)**

b. Find n^{th} derivation of $\dfrac{x}{(2x+1)(3x-1)}$. **(05 Marks)**

c. Find the radius of curvature of the curve $x^3 + y^3 = 3axy$ at $\left(\dfrac{39}{2}, \dfrac{39}{2}\right)$ **(05 Marks)**

OR

2 a. $y = a\cos(\log x) + b\sin(\log x)$, prove that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)g_n = 0$. **(06 Marks)**

b. Prove that the following pairs of polar curves intersect orthogonally $r = a(1+\cos\theta)$, $r = b(1-\cos\theta)$. **(05 Marks)**

c. For the curve $y = \dfrac{ax}{a+x}$ where a is a constant, prove that $\left(\dfrac{2\rho}{a}\right)^{2/3} = \left(\dfrac{y}{x}\right)^2 + \left(\dfrac{x}{y}\right)^2$. **(05 Marks)**

Module-2

3 a. $u = \log(x^3 + y^3 + z^3 - 3xyz)$, prove that :

(i) $\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} + \dfrac{\partial u}{\partial z} = \dfrac{3}{x+y+z}$ (ii) $\left(\dfrac{\partial}{\partial x} + \dfrac{\partial}{\partial y} + \dfrac{\partial}{\partial z}\right)^2 = \dfrac{9}{(x+y+z)^2}$ **(06 Marks)**

b. Evaluate $\lim\limits_{x\to 0}\left(\cot^2 x - \dfrac{1}{x^2}\right)$. **(05 Marks)**

c. Expand $f(x) = \log\cos x$ in powers of $\left(x - \dfrac{\pi}{3}\right)$ upto the Fourth degree term. **(05 Marks)**

OR

4 a. $u = x + y + z$, $uv = y + z$, $z = uvw$, show that $\dfrac{\partial(x,y,z)}{\partial(u,v,w)} = u^2 v$. **(06 Marks)**

b. $z = f(x,y)$ and $x = e^u + e^{-v}$ and $y = e^{-u} - e^v$ show that $\dfrac{\partial z}{\partial u} - \dfrac{\partial z}{\partial v} = x\dfrac{\partial z}{\partial x} - y\dfrac{\partial z}{\partial y}$. **(05 Marks)**

c. Find Maclaurin's series expansion for $\log\sec x$ upto 4^{th} degree. **(05 Marks)**

Module-3

5 a. A particle moves along the curve $x = a(3t - t^3)$, $y = 3at^2$, $z = a(3t + t^3)$ where a is a constant. Find the components of velocity and acceleration vectors at t = 1 along the vector $\vec{c} = \hat{i} - \hat{j} + 2\hat{k}$. **(06 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

b. Find the constants a, b, c so that the vector field $\vec{f} = (x+2y+az)\hat{i} + (bx-3y-z)\hat{j} + (4x+cy+2z)\hat{k}$ is irrotational. **(05 Marks)**

c. Prove that $\operatorname{div}(\operatorname{curl}\vec{t}) = 0$. **(05 Marks)**

OR

6 a. If $\vec{v} = \vec{w} \times \vec{r}$ where $\vec{w}$ is a constant vector, show that $\vec{w} = \frac{1}{2}\operatorname{curl}\vec{v}$. **(06 Marks)**

b. If $\vec{f} = \operatorname{grad}(x^3y + y^3z + z^3x - x^2y^2z^2)$, find $\operatorname{div}\vec{f}$ at (1, 2, 3). **(05 Marks)**

c. Show that $\vec{f} = \frac{x\hat{i} + y\hat{j}}{x^2 + y^2}$ is irrotational. **(05 Marks)**

Module-4

7 a. Obtain the reduction formula for $\int \cos^n x dx$ and evaluation $\int_0^{\pi/2} \cos^n x dx$. **(06 Marks)**

b. Solve $y \sin 2x dx - (1 + y^2 + \cos^2 x)dy = 0$. **(05 Marks)**

c. Find the orthogonal trajectories of the family of curve $y^2 = cx^3$ where c is a constant. **(05 Marks)**

OR

8 a. Evaluate $\int_0^{\pi/6} \cos^4 3\theta \sin^3 6\theta d\theta$. **(06 Marks)**

b. Solve $2\frac{dy}{dx} - y\sec x = y^3 \tan x$. **(05 Marks)**

c. Find the orthogonal trajectories of the family of curves $r = a\cos^2\frac{\theta}{2}$ where a is a parameter. **(05 Marks)**

Module-5

9 a. Solve the following system of equations by Gauss-Elimination method:
$x + y + z = 9$, $x - 2y + 3z = 8$, $2x + y - z = 3$ **(06 Marks)**

b. By using power method to find the largest eigen value and the corresponding eigen vector of the matrix. Taking $[1, 0, 0]^1$ as a initial eigen vector.

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$ **(05 Marks)**

c. Find the inverse transformation of the following linear transformation:
$y_1 = x_1 + 2x_2 + 5x_3$, $y_2 = 2x_1 + 4x_2 + 11x_3$, $y_3 = -x_2 + 2x_3$ **(05 Marks)**

OR

10 a. Solve the following system of equations by Gauss Seidel Method
$5x + 2y + z = 12$, $x + 4y + 2z = 15$, $x + 2y + 5z = 20$
Carryout 4 iterations. Taking the initial approximation to the solution as (1, 0, 3). **(06 Marks)**

b. Diagonalize the matrix $\begin{bmatrix} -19 & 7 \\ -42 & 16 \end{bmatrix}$. **(05 Marks)**

c. Reduce the following quadratic form $x_1^2 + 3x_2^2 + 3x_3^2 - 2x_2x_3$ to canonical form. **(05 Marks)**

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